New technique for measuring the dynamic Young's modulus between 295 and 6 K

J. Zhang*, A. Nyilas and B. Obst

Karlsruhe Nuclear Research Centre, Institute of Technical Physics, Postfach 3640, W-7500 Karlsruhe, Germany

A transducer system based on strain gauge technology was developed and applied to bar type specimens to determine the dynamic Young's modulus. The system was installed in a continuous flow cryostat working in the temperature range 295–6 K. The adapted technique is the remote mechanical excitation of the bar as a cantilever beam and the on-line determination of the fundamental free vibration frequency using a commercially available electronic instrument. This simple and economic method is able to measure the material's dynamic Young's modulus, E, and shear modulus, G, with a high degree of accuracy (error $< \pm 2.0\%$). The Poisson's ratio, ν, can therefore be determined, according to the measured values of E and G. Six different engineering materials were characterized with respect to their elastic properties between 295 and 6 K. The dynamic Young's modulus *versus* temperature measurements were compared with the results of static stress–strain measurements carried out in the same test facility.

Keywords: Young's modulus; cryostats; strain gauges

Knowledge of the Young's modulus is essential for elastic stress–strain analysis of solid structures under operational loads and engineering design of cryogenic load bearing components requires reliable data concerning the temperature dependence of the Young's modulus. For engineering alloys collected data exist in various reports[1,4], which are currently used in the design of cryogenic machines. However, information concerning the temperature dependence of Young's modulus is lacking in certain cases, as for example for most weld metals. To date a few ultrasonic measurements at 295 K along the cross-section of a weld metal have been reported[5,6], which give only a rough idea of the variation in Young's modulus along the weldment due to the required large specimen diameter (> 8 mm). For heavily loaded, large welded structures knowledge of the elastic properties of the weld metal is absolutely necessary for structural mechanics analysis.

This paper presents a new transducer, based on strain gauge technology, capable of measuring Young's modulus from 295 down to 6 K. This dynamic measurement technique comprises the determination of the fundamental frequency of a free vibrating specimen in the form of a cantilever beam. The transducer itself is the strain gauge bonded specimen. The strain gauges placed on the surface of the specimen, combined to a full bridge circuit, transmit an electrical a.c. signal, corresponding to the free vibration of the specimen, to a commercially available electronic device which measures the system's frequency. Besides measurement of Young's modulus, E, it was also possible to determine the temperature dependent elastic constants (shear modulus, G, and Poisson's ratio, ν) directly using the same technique. Due to the use of thin bar specimens ($\approx 120 \times 10 \times 1$ mm) this measurement technique also makes it possible to determine the anisotropic elastic properties along the cross-section of the weld metal.

The temperature dependence of Young's modulus, E, between 295 and 6 K is reported for six engineering materials, commonly designated Ti-6Al-4V, ETP-copper, AlMg3, AISI 304, AISI 316 LN and pultruded GFRP. Measurements on the GFRP were made perpendicular to the pultrusion direction, giving the flexural Young's modulus. The elastic constants, G and ν, were also measured for the two materials, AISI 304 and AlMg3. A slight decrease in the Young's modulus could be observed at low temperatures for some of the metallic materials tested. In addition, the Young's modulus of the AISI 316 LN was determined with a tensile specimen between 295 and 7 K. An interpretation of the difference between the dynamic and static measurement methods is given below. The data obtained for the ETP-copper show a discrepancy below ≈ 80 K with results from the

*Guest scientist from the General Research Institute for Non-Ferrous Materials (GRINM), Beijing, China

existing cryogenic material data base. As the present work is biased towards the new measurement technique, details of these findings (for the ETP-copper) will be discussed after completing additional tests in the near future.

Test method
Transducer system

The new sensing unit for determination of the temperature dependent Young's modulus, E, is the specimen itself, gripped in a fixture and acting as a cantilever beam. Four strain gauges were placed on the surface of the beam, two on each side, and were combined with a full bridge circuit. The specimen dimensions were optimized with respect to a high signal-to-noise ratio and were typically in the range of $100 \times 10 \times 1$ mm. After mechanical excitation the free vibration state of the beam was transmitted by the output signals of the strain gauge system to a commercial electronic device[7] (Grindo-Sonic, type MK 4). The strain gauge analogue signals coming from the excited beam were analysed electronically by the Grindo-Sonic device and the time interval between two periods was displayed in microseconds. The bridge voltage of the strain gauge excitation was between 0.5 and 1.0 V.

The strain gauge technique offers advantages related to its excellent working capability in a low temperature environment compared with commercial transducers such as microphone or quartz systems. Quartz transducers need specimens of a larger volume and also involve an elaborate technique for specimen/transducer attachment. The microphone, on the other hand, can be rejected for operation in environments such as that presently considered (helium gas of variable pressure at low temperatures).

Low temperature application

For the cryogenic measurements and the necessary mechanical excitation of the cantilever beam an existing dynamic test facility was used. This test facility (*Figures 1* and *2*) comprises a continuous flow cryostat with a test chamber with dimensions 265 (diameter) $\times$ 210 mm.
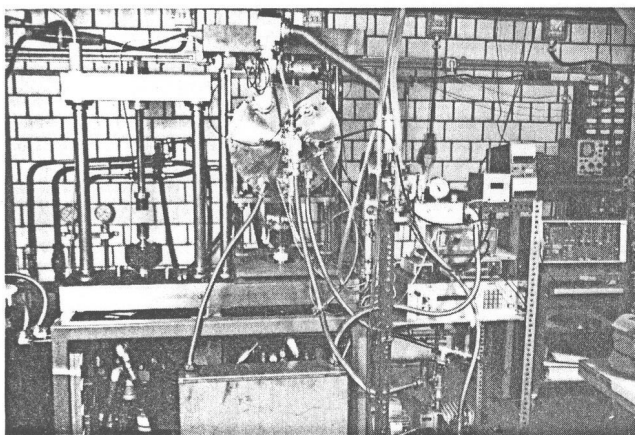

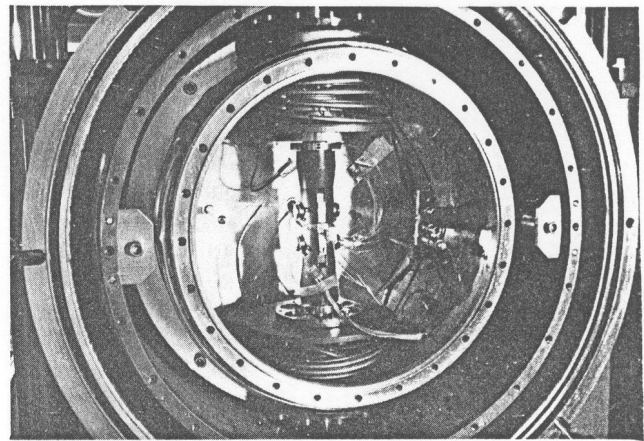**Figure 1** Cryogenic dynamic loading test facility

Figure 2 Inside view of the cryogenic test chamber of the helium continuous flow cryostat

Two pull rods, entering the cryostat through a system of internal and external stainless steel bellows, allow mechanical investigations at low temperatures. A hydraulic servocylinder of ± 25 kN capacity (MTS, model 810), programmable with a microprocessor unit, is able to generate any type of stroke movement. With this arrangement it was possible to excite the specimen (fixed to the upper rod of the machine) with a simple spring system. The temperature stability of the chamber could be attained at ± 0.2 K in the low temperature regime (<60 K). The helium exchange gas had a pressure of 1.1 bar* at 295 K and 27 mbar at 7 K, respectively.

Mechanical analysis

For a simple, supported, bar type beam (beam resting on two soft supports) the Young's modulus has been shown to depend on the elastic vibration of the specimen

$$E = K'K_p \frac{\Upsilon L^4}{a^2} f^2 \times 10^{-10} \tag{1}$$

where: L = length of the beam (in cm); a = thickness of the beam (in cm); Υ = specific density (in g cm^{-3}); f = frequency of the fundamental vibration (in Hz); K_p = size factor given below by Equation (2); K' = 0.946 (a temperature independent constant); and E = Young's modulus (in GPa).

$$K_p = 1 + \frac{a^2}{L^2}\left(7.32 - 11\frac{a^2}{L^2}\right) \tag{2}$$

For a cantilever beam, Equation (1) also holds in principle, but with another numerical value for K'. In this work K' was determined for straightforward cantilever beam type, supported bar specimens. A long bar of $200 \times 10 \times 1$ mm was accurately machined using an electro-discharge method (the AISI 304 material). The Young's modulus was determined at 295 K using the commercial microphone transducer and Equation (1).

*1 bar = 10^5 N m^{-2}

For this measurement the bar was resting on two soft supports and the mechanical excitation was performed according to the instructions provided with the Grindo-Sonic device. After this measurement the same bar was supported as a cantilever beam in a test fixture with a certain span length. The particular frequencies for different span lengths, substituted in Equations (1) and (2) with the known Young's modulus, E, give a new numerical value for K'. According to this method, K' was determined to be 39.2 with a numerical error $<0.1\%$ with respect to different readings.

After establishing the new constant, K', for the case of a cantilever beam, the specimen dimensions were optimized with respect to the strain gauge application. A thin bar type specimen (thickness <1 mm) seemed to be well excited using the existing mechanical arrangement. Four strain gauges (TML Co., type CFLA, 350 Ω) were glued with a cryogenic working adhesive ≈ 10 mm apart, from the supported end. Two of the strain gauges were positioned on one flat surface of the beam, to supply compression and tension signals during the free vibration. The signals from the bridge were amplified with a commercial 5 kHz modulated amplifier. The height of the output signals during the free vibration was $1-2$ V. Frequencies determined using the strain gauges as sensing units showed little difference from the numerical value obtained with the microphone as transducer. So, to a good approximation, for further evaluations the 295 K value of the Young's modulus, E, determined with the microphone as a transducer, was used.

According to the relation

$$E_T = \left(\frac{f_T}{f_{295}}\right)^2 E_{295} \tag{3}$$

the temperature dependence of the Young's modulus, E, can be calculated in a straightforward manner, where: f_T = frequency at temperature T (determined by the strain gauge system); f_{295} = frequency at 295 K (determined by the strain gauge system); E_{295} = Young's modulus at 295 K (determined with the microphone); and E_T = Young's modulus at temperature T.

Poisson's ratio

For determination of Poisson's ratio the fundamental equation

$$\nu = \frac{E}{2G} - 1 \tag{4}$$

can be used by measuring E and G simultaneously for one specimen. The shear modulus, G, is measured by using the equation valid for bar type specimens

$$G = F\left(\frac{a}{b}\right)\Upsilon L^2 f^2 \times 10^{-10} \tag{5}$$

and following the instructions provided with the Grindo-Sonic device. *Table 1* gives the numerical values of the

Table 1 Function $F(a/b)$ [of Equation (5)] valid for bar type specimens

a/b	$F(a/b)$
1.00	4.7394
1.20	4.9203
1.40	5.2888
1.60	5.8246
1.80	6.4956
2.00	7.2820
2.50	9.6856
3.00	12.655

function $F(a/b)$ for bar type specimens (a and b are the width and thickness of the bar, respectively).

For these measurements a block of the material to be tested was machined to typical dimensions of $20 \times 40 \times 100$ mm. By determining the transversal and torsional vibrational frequencies at 295 K with the Grindo-Sonic device, the G and E values of the materials were established. An additional specimen in the form of a flat sheet of the same material was prepared, with dimensions $70 \times 40 \times 1$ mm, to give a cantilever beam unit in the cryostat. Two sensing units based on a full bridge combination were positioned on the flat surface of this specimen (*Figure 3*). The position of the glued strain gauges was chosen such that each sensing system transmits either the longitudinal vibration (E) or the torsional vibration (G) to the amplifier. The non-symmetric excitation point leads to torsional and longitudinal beam vibrations.

Again, using Equation (3) with the torsional and longitudinal frequencies the temperature dependent E and G values of the material were determined. Plotting E and G values as a function of temperature, a smooth line could be drawn from 295 to 6 K. The Poisson's ratio, ν, was evaluated using the determined E and G functions and Equation (4).

Mechanical measurements

Conventional stress—strain type experiments were performed to compare the dynamic and static data in the
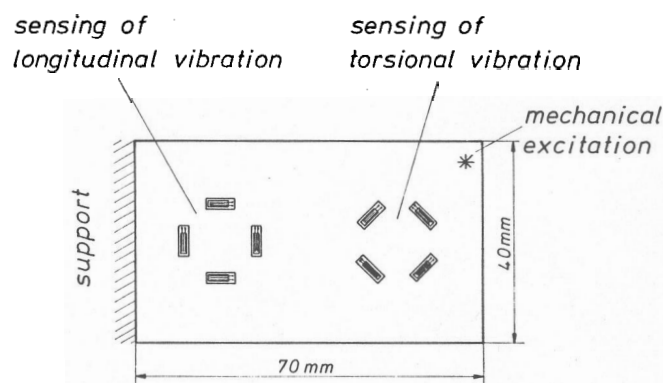

Figure 3 Two full bridge strain gauge sensing combination positioned on the surface of the cantilever beam plate (strain gauge dimensions 1×1.5 mm)

cryogenic regime. The static measurements were carried out with cylindrical specimens of 100 mm length. The reduced length was 80 mm with a section diameter of 5 mm. These tests were conducted in the same helium flow cryostat as described above using a high resolution extensometer of high linearity ($\pm 0.2\%$) and high sensitivity (2.3 mV per V for full deflection of 1 mm). The extensometer was calibrated twice prior to the tests in the same cryostat. Two measurements with similar specimens of the same material were taken as the standard case. The difference between the two tests was below $\pm 2\%$. The average of the two recorded values was taken as the reference value.

Materials

The investigated materials, their specific parameters and the measured frequencies of the specimens at 295 K are given in *Table 2*. The AISI 316 LN was available in two heat types. The first heat type, which was used for the cryogenic dynamic measurements, was the cold worked sheet of Werkstoff No. 1.4439 with $\approx 5\%$ molybdenum. The second heat type was a 30 mm thick plate of Werkstoff No. 1.4429 with $\approx 2.7\%$ molybdenum; the static measurements were performed with this material.

The 295 K frequencies in *Table 2* refer to the case of the specimens measured as a simple supported beam before strain gauge bonding. This precaution was taken to avoid the effect of the strain gauges on the free vibration. In this case, the sensing unit was the microphone. After strain gauge application the cantilever beam specimen, clamped in the test fixture, acted as a transducer. The mean of six readings taken after each individual mechanical excitation was used for calculating the Young's modulus, E, at the particular temperature.

Results

The temperature dependence of the Young's modulus, E, was determined for the materials given in *Table 2*. The evaluation was performed using Equation (3) without taking into account the effects of thermal contraction. *Figures 4, 5* and *6* show the Young's modulus, E, *versus* temperature for the six different engineering materials investigated. Some of the metallic materials show a slight decrease in the Young's modulus at low temperatures (< 30 K). The anomalous change in the Young's modulus of the AISI 304 below 80 K can

Figure 4 Temperature dependence of Young's modulus of AISI 304 and AISI 316 LN between 300 and 6 K

Table 2 295 K Young's modulus of different engineering materials determined by dynamic vibration method using the virgin specimen as a simple supported beam

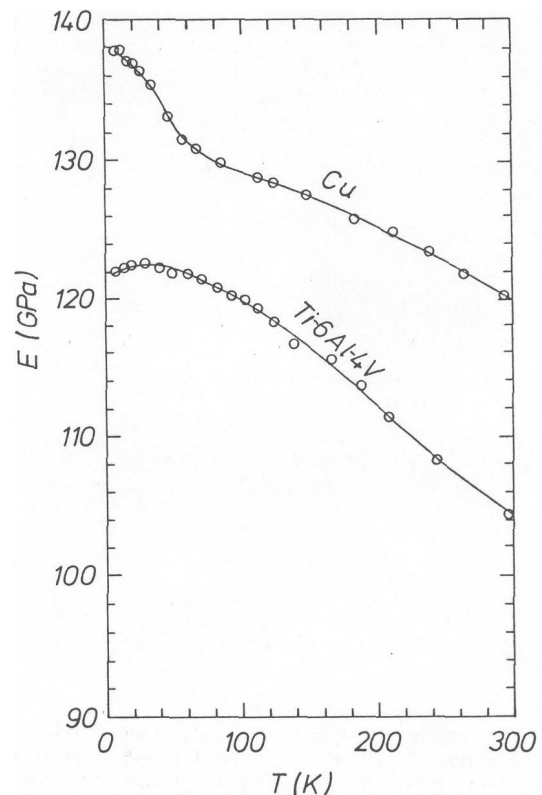| Material	Span length (mm)	Thickness (mm)	Density (g cm^{-3})	Frequency (Hz)	Young's modulus (GPa)
Ti-6Al-4V	100.5	1.470	4.46	112.78	105.1
AlMg3	68.9	1.000	2.70	174.25	72.5
ETP-copper	65.4	1.020	8.96	138.58	118.8
304	75.9	1.020	7.96	143.43	203.7
316 LN	42.4	0.795	7.88	335.40	177.7
GFRP	77.8	3.090	2.01	394.71	47.6

Figure 5 Temperature dependence of Young's modulus of Ti-6Al-4V and ETP-copper between 300 and 6 K
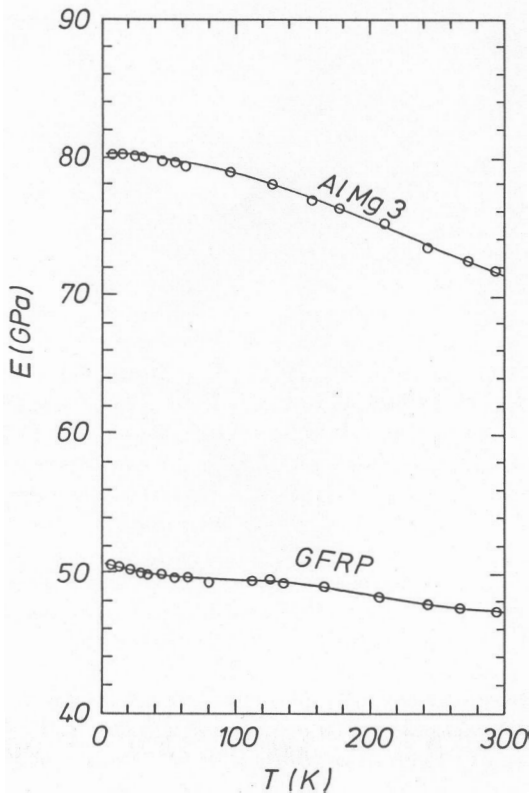
Figure 6 Temperature dependence of Young's modulus of AlMg3 and pultruded GFRP between 300 and 6 K
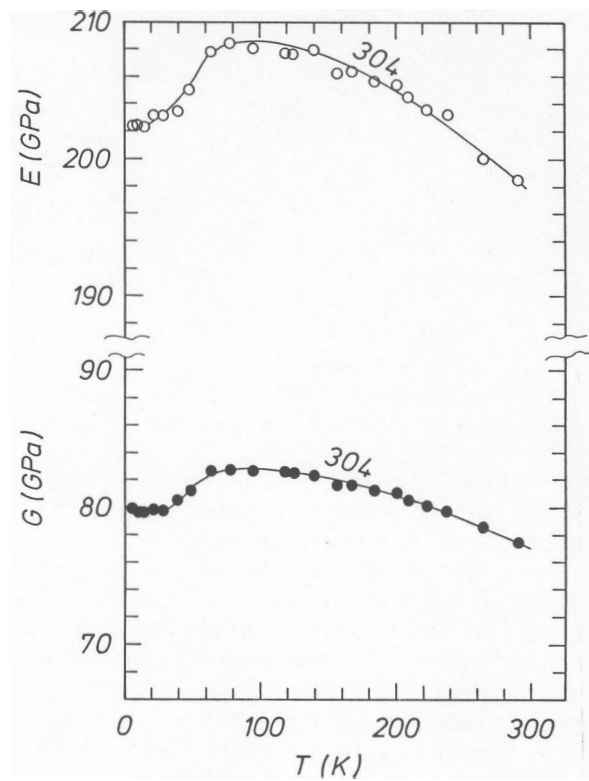
Figure 8 Temperature dependence of shear modulus and Young's modulus of AISI 304 between 300 and 6 K
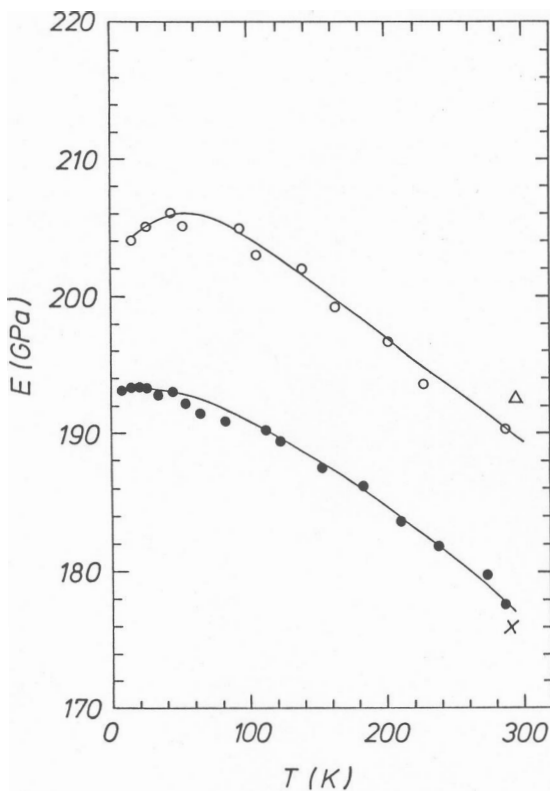
Figure 7 Static and dynamic Young's modulus results for two different heat types of AISI 316 LN and their temperature dependence between 300 and 6 K. △, Dynamic test of AISI 316 LN with 2.7% Mo; O, static tensile test of AISI 316 LN with 2.7% Mo; ●, dynamic test of AISI 316 LN with 5% Mo; X, static tensile test of AISI 316 LN with 5% Mo

be attributed to a reversible second order magnetic transition (as described in Reference 1) (*Figure 4*). *Figure 5* shows the values for ETP-copper. An increase in the Young's modulus occurs at ≈ 70 K according to the dynamic measurements. Data in the literature[8,9] for Young's modulus *versus* temperature for annealed copper also show a slight indication of a similar increase below 70 K, if one carefully considers the experimentally obtained values from different authors[8-11].

Figure 7 gives the results with AISI 316 LN for the two different heat types. The difference between the static tensile test and the dynamic measurements is within 1% at 295 K. It is found that the dynamically determined Young's modulus is always higher than the statically obtained value. The heat to heat difference is attributed to the difference in chemical composition.

According to the present results, some of the investigated metallic materials show a decrease in Young's modulus below a characteristic temperature. Regarding the fact that mechanical excitation of the cantilever beam produces an estimated strain in the range 0.05−0.1% at the outer fibres of the plate, it is easily imaginable that this strain is in the range of microplasticity. As stated elsewhere[12,13], at low temperatures a change of the dislocation character from screw to edge occurs in fcc metals and alloys leading to a more or less steep increase in the yield strength. Therefore, it is assumed that the change in the dislocation character may well influence the vibration properties of the specimen due to the mechanical nature of such measurements.

Figure 8 gives the shear modulus, *G*, and Young's modulus, *E*, results obtained between 295 and 6 K with the AISI 304. The difference in the values of Young's
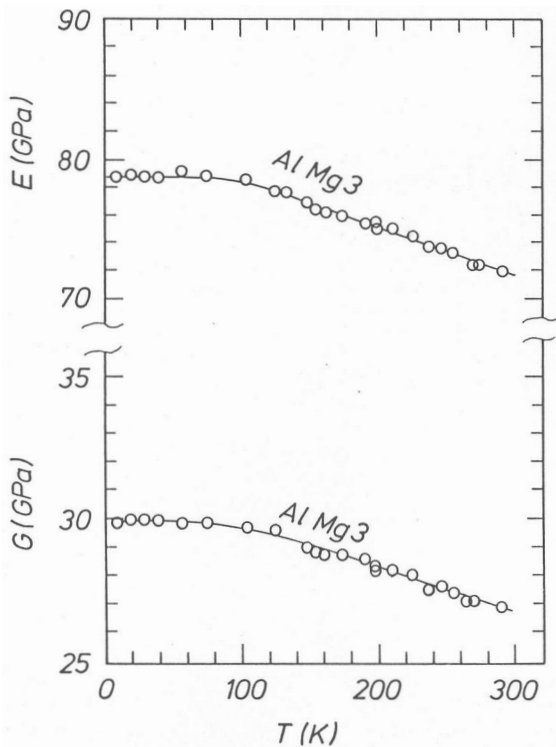
Figure 9 Temperature dependence of shear modulus and Young's modulus of AlMg3 between 300 and 6 K
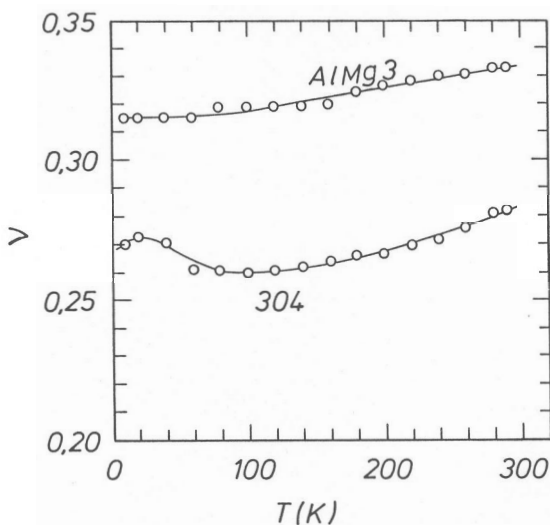

Figure 10 Poisson's ratio, ν, of AISI 304 and AlMg3 between 300 and 6 K

modulus from this investigation compared with the findings in *Figure 4* (2−4%) may be attributed to the heat to heat variation, which is obviously responsible for such phenomena. In *Figure 9* the shear modulus and the

Young's modulus *versus* temperature measurements are given for the AlMg3. The Poisson's ratio, ν, calculated according to Equation (4) is given for both these materials in *Figure 10*.

Conclusions

A new technique based on strain gauge technology was developed for measuring the dynamic elastic constants, E, G and ν, between 295 and 6 K. The engineering materials AISI 316 LN, AISI 304, Ti-6AL-4V, AlMg3, pultruded GFRP and ETP-copper were investigated with respect to their Young's modulus temperature dependence. For AISI 304 and AlMg3 the elastic constants, G and ν, were also measured using the same method. The technique presented allows determination of the temperature dependent Young's modulus with small (≈ 1 mm thickness) specimens and can be qualified for sectional analysis to characterize the anisotropic elastic behaviour of heavy weldments.

References

1   **Ledbetter, H.M., Weston, W.F. and Naimon, E.R.** Low-temperature elastic properties of four austenitic stainless steels *J Appl Phys* (1975) **46**(9) 3855−3860
2   **Ledbetter, H.M.** Temperature behaviour of Young's modulus of forty engineering alloys *Cryogenics* (1982) **22** 653−656
3   **Ledbetter, H.M., Frederick, N.V. and Austin, M.W.** Elastic-constant variability in stainless-steel 304 *J Appl Phys* (1980) **51**(1) 305−309
4   **Ledbetter, H.M.** Stainless-steel elastic constants at low temperatures *J Appl Phys* (1981) **52**(3) 1587−1589
5   **Kimmins, S.T. and Horton, C.A.P.** Anisotropy of elastic and tensile properties in type 17 Cr−8Ni−2Mo weld metal welding *Nuclear Engineering: Lectures of the 4th International Conference in Aachen* (1982) DVS 75, 280−284
6   **Kapranos, P.A., Al-Helaly, M.M.H. and Whittaker, V.N.** Ultrasonic velocity measurements in 316 austenitic weldments *Brit J NDT* (1981) **23**(6) 288
7   **Grindo-Sonic** *Advanced Mat Processes* (1990) **138** (issue 3, Sept) 6
8   **Biller, E. and Helow, R.** Uber die Temperaturabhängigkeit des komplexen Elastizitätsmoduls von Kupfer *Zeitschrift für Physik* (1969) **224** 102−112
9   **Stokes, H.J.** Apparatus for the measurement of Young's modulus, between −200 and 700°C by transverse vibration in vacuum *J Sci Instrum* (1960) **37** 117−120
10 **Ledbetter, H.M. and Naimon, E.R.** Elastic properties of metals and alloys:II. Copper *J Phys Chem Ref Data* (1974) **3** 897−935
11 **Simon, N.J. and Reed, R.P.** Cryogenic properties of copper and copper alloys, preliminary draft of report from the Fracture and Deformation Division, National Bureau of Standards, Boulder, Colorado, USA (1987)
12 **Seeger, A.** *Dislocations and Mechanical Properties of Crystals* (Eds Fisher, J.C., Johnston, W.G., Thomson, R. and Vreeland, T. Jr) John Wiley & Sons, New York, USA (1957) 243
13 **Obst, B. and Nyilas, A.** Experimental evidence on the dislocation mechanism of serrated yielding in FCC metals and alloys at low temperature *Mat Sci Eng* in press A137 (1991) 141−150